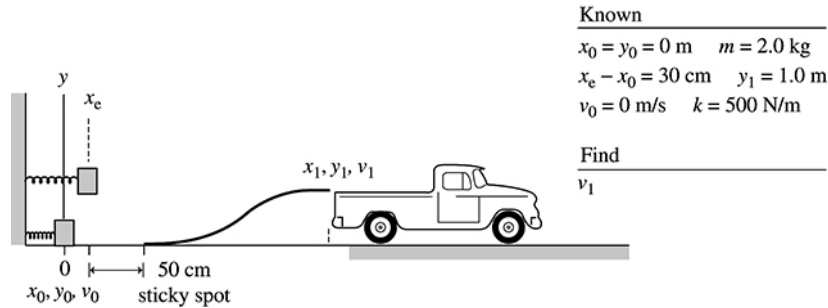


**11.49. Model:** We will use the spring, the package, and the ramp as the system. We will model the package as a particle.

**Visualize:**



We place the origin of our coordinate system on the end of the spring when it is compressed and is in contact with the package to be shot.

**Model:** (a) The energy conservation equation is

$$K_1 + U_{g1} + U_{s1} + \Delta E_{th} = K_0 + U_{g0} + U_{s0} + W_{ext}$$

$$\frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}k(x_e - x_e)^2 + \Delta E_{th} = \frac{1}{2}mv_0^2 + mgy_0 + \frac{1}{2}k(\Delta x)^2 + W_{ext}$$

Using  $y_1 = 1 \text{ m}$ ,  $\Delta E_{th} = 0 \text{ J}$  (note the frictionless ramp),  $v_0 = 0 \text{ m/s}$ ,  $y_0 = 0 \text{ m}$ ,  $\Delta x = 30 \text{ cm}$ , and  $W_{ext} = 0 \text{ J}$ , we get

$$\frac{1}{2}mv_1^2 + mg(1 \text{ m}) + 0 \text{ J} + 0 \text{ J} = 0 \text{ J} + 0 \text{ J} + \frac{1}{2}k(0.30 \text{ m})^2 + 0 \text{ J}$$

$$\frac{1}{2}(2.0 \text{ kg})v_1^2 + (2.0 \text{ kg})(9.8 \text{ m/s}^2)(1 \text{ m}) = \frac{1}{2}(500 \text{ N/m})(0.30 \text{ m})^2$$

$$\Rightarrow v_1 = 1.70 \text{ m/s}$$

(b) How high can the package go after crossing the sticky spot? If the package can reach  $y_1 \geq 1.0 \text{ m}$  before stopping ( $v_1 = 0$ ), then it makes it. But if  $y_1 < 1.0 \text{ m}$  when  $v_1 = 0$ , it does not. The friction of the sticky spot generates thermal energy

$$\Delta E_{th} = (\mu_k mg)\Delta x = (0.30)(2.0 \text{ kg})(9.8 \text{ m/s}^2)(0.50 \text{ m}) = 2.94 \text{ J}$$

The energy conservation equation is now

$$\frac{1}{2}mv_1^2 + mgy_1 + \Delta E_{th} = \frac{1}{2}k(\Delta x)^2$$

If we set  $v_1 = 0 \text{ m/s}$  to find the highest point the package can reach, we get

$$y_1 = \left(\frac{1}{2}k(\Delta x)^2 - \Delta E_{th}\right) / mg = \left(\frac{1}{2}(500 \text{ N/m})(0.30 \text{ m})^2 - 2.94 \text{ J}\right) / (2.0 \text{ kg})(9.8 \text{ m/s}^2) = 0.998 \text{ m}$$

The package does not make it. It just barely misses.